A constant-force bistable mechanism for force regulation and overload protection

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Abstract

A novel constant-force bistable mechanism (CFBM) allowing constant contact force and overload protection is developed. When a device equipped with a CFBM is loaded by an unknown force exceeding a critical force of the CFBM, the CFBM can snap to its other stable equilibrium state to safeguard the device. The bistability of the mechanism originates from combined compression and bending of the beam structures. Finite element analyses are used to characterize the constant-force behavior and bistability of the mechanism under static loading. A design formulation is proposed to find the CFBM shape for a specified displacement range with constant output force of the mechanism. Prototypes of the CFBM are fabricated and tested. The characteristics of the CFBM predicted by theory are verified by experiments. Using the CFBM, sophisticated sensors and control system for force regulation of machining systems can be eliminated.

Keywords: Bistable; constant-force mechanism; overload protection.

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1. Introduction

Responding to the increasing needs in many systems where a variable output force is undesirable, mechanisms which provide a near-constant output force over a prescribed deflection range have been developed and are defined as constant-force mechanisms (CFMs). These mechanisms have been gaining more and more attention in recent years [1-5]. CFMs can be designed for concrete testing equipment [1], exercise machines [2] and electrical contacts [3]. A constant-force actuator based on dielectric elastomers is proposed by Berselli et al. [4] for applications in robotics and mechatronics. Lan et al. [5] developed a compliant CFM for force regulation of robot end-effectors operating in an unknown environment.

CFMs can be utilized in systems to reduce the need for complex control algorithms and feedback loops [6, 7]. Without sophisticated control systems, CFMs made of passive mechanisms may minimize control effort and reduce cost without losing precision. CFMs have been developed by various investigators [8-12]. Pedersen et al. [8] used topology and size optimization to design a transmission mechanism which converts the constant stiffness of an actuator into constant output force. Weight et al. [9] proposed a CFM composed of a bent beam and a cam for electrical contacts in personal digital assistant (PDA) dock stations to improve the reliability of high-cycle electrical connectors. Their beam and cam combination provides the strain energy storage device necessary for constant force behavior. Boyle et al. [10] presented a CFM consisting of a rigid link, a flexible segment and a slider. His compliant CFM offers the possibility of a new type of spring element for a variety of applications. Nahar and Sugar [11] designed a double-slider CFM, where two springs are attached to the sliders. They also proposed the design of a micro compliant slider-crank CFM. However, in order to facilitate these

mechanisms in microelectromechanical system (MEMS) devices, monolithic compliant mechanisms which can be fabricated by microfabrication technologies are favored over the macro devices. With fewer movable joints, use of compliant mechanisms results in reduced wear, reduced need for lubrication, and an increased mechanism precision [12].

When integrated with a bistable mechanism, the CFM can have the potential applications in force regulation and overload protection. Here, the CFM with the added function of bistability is termed as a constant-force bistable mechanism (CFBM). In some applications such as micro-ultrasonic machining, a variation in contact force would cause extreme tool wear and worsen the micromachining accuracy, and the accidental overloading could break the tool easily [13]. The incorporation of CFBM into these micro machining stations could help to regulate the output force to accommodate for the variation in surface roughness and preserve a stable contact force. When the micro tool is suddenly loaded by an unknown force exceeding a critical value of the CFBM, the CFBM can snap to its other stable equilibrium state to safeguard the device. Without resorting to force sensors, controllers and actuators, the CFBM can be effective to protect the micro machining system from surface variation of unknown/harsh environment.

This paper describes a design of a compliant CFBM. The bending of elastic beams of the mechanism is exploited to provide the constant output force over a range of input displacements. Bistability is provided by buckling of curved-beam structures of the mechanism. The design is based on the compliant bistable mechanisms reported by Wang et al. [14] and Qiu et al. [15], where the bistability originates from combined compression and bending of a chevron-type mechanism consisting of straight beams and flexible hinges [14] or of a double curved-beam structure with carefully selected geometries [15]. The proposed CFBM has a curved-beam structure embedded in a chevron-type mechanism. An optimization method is used to design the CFBMs. Finite element analyses are carried out to evaluate the mechanical behaviors of the design obtained by the optimization procedure. Prototypes of the device are fabricated using a milling process. Experiments are performed to demonstrate the effectiveness of the CFBM.



Fig. 1 Operational principle.

2. Design

2.1 Operational principle

A schematic of a CFBM is shown in Fig. 1(a). A Cartesian coordinate system is also shown in the figure. The mechanism is a compliant chevron-type mechanism consisting of a shuttle mass, a guide beam, curved beams and lateral beams. The joint of upper lateral beam, lower lateral beam and outer curved beam acts similar to a flexible hinge and facilitates the rotation of the CFBM. The assembly of the upper lateral beam, lower lateral beam and outer curved beam is termed as "flexible hinge." The guide beam is employed to prevent the mechanism from twisting during operation, and is designed to be stiff. Upon the application of a force F to the shuttle mass, the lateral beams and outer curved beams deflect initially, storing the strain energy. As the CFBM deflects further, the bending energy in the flexible hinges and inner curved beams increases. While the compression energy in the inner curved beams increases to a maximum at a certain displacement of the mechanism, critical displacement of the CFBM, but then decreases; the mechanism snaps towards its second stable equilibrium state. Fig. 1(a) and (b) show the first and second stable equilibrium state of the mechanism, respectively.



Fig. 2 (a) A typical force versus displacement curve of the CFBM and the corresponding positions at displacement a (b), displacement c (c), displacement d (d), and displacement g (e).

If the mechanism is designed so that the majority of the strain energy is absorbed by the flexible hinges and little strain energy is stored in the inner curved beams before it snaps to the second stable state, where the strain energy level of the inner curved beams is nearly constant and the CFBM has the characteristics of nearly constant output force over a range of displacement. Fig. 2(a) shows a typical output force versus displacement (f- δ) curve of the CFBM. Fig. 2(b-e) illustrates a four-step operation of the mechanism. First, a force *F* is applied to the mechanism through the shuttle mass (see Fig. 2(b)) to deflect the flexible hinge. When the deflection of the shuttle mass is within the interval of $\delta = b$ to $\delta = c$, the mechanism generates a nearly constant output force over this range of displacement, hence defined as operational range, D (see Fig. 2(a)). The maximum output force f_d occurs at the displacement $\delta = d$ (see Fig. 2(d)). f_e is the minimum output force of the CFBM and has a negative value (see Fig. 2(a)). When the force applied to the mechanism *F* is greater than f_d , the mechanism moves towards its second stable position g (see Fig. 2(e)), where the output force of the CFBM is zero. The CFBM can not only serve as a transmission mechanism downstream of a device if the device is to be used in constant force applications [8], but also provide a simple means of protection of the device in the event of mechanical shock/sudden overloading.





Fig. 3 (a) A schematic of the upper-left branch of the CFBM. (b) Dimensions of the guide beam and the shuttle mass.

2.2 Design

The design of the CFBM is based on an optimization procedure where the shape of the lateral beams and curved beams is optimized via the parameters of a cubic Bézier curve and a cosine curve, respectively. Fig. 3(a) is a schematic of the upper-left branch of the CFBM. The cubic Bézier curve is determined by a four-point Bézier polygon $B_{u1}B_{u2}B_{u3}B_{u4}$ for the upper lateral beam as shown in Fig. 3(a). As described by Rogers and Adams [16], the first and last points, B_{u1} and B_{u4} , respectively, on the curve are coincident with the first and last points of the defining polygon. The tangent vectors at the ends of the curve have the same directions as the first and last polygon spans, respectively. The parametric cubic Bézier curve for the upper lateral beam is given by [17]

$$P(t) = \begin{bmatrix} (1-t)^3 & 3t(1-t)^2 & 3t^2(1-t) & t^3 \end{bmatrix} \begin{bmatrix} P_{u1} \\ P_{u2} \\ P_{u3} \\ P_{u4} \end{bmatrix} \quad 0 \le t \le 1$$
(1)

where t is the parameter, P is the position vector of a point on the curve, and P_{ui} is the position vector of the point B_{ui} . The shape of the upper lateral beam is optimized by allowing points B_{u2} and B_{u3} to move in the space enclosed by the dotted quadrilateral $B_{u1}B_{u4}AC$ in Fig. 3(a). The position of the point B_{u1} is fixed, and the Cartesian coordinates (x, y) of the points B_{u2} , B_{u3} and B_{u4} are design variables. The shape of the lower lateral beam is optimized similarly to the upper lateral beam.

The shape of the curved beams is

$$y_r = \frac{h_i}{2} \left(1 - \cos \frac{\pi x_r}{L_i} \right) \tag{2}$$

where (x_r, y_r) is the position vector with the reference origin at the left end of the curved beams. *L* and *h* are the span and apex height of the curved beams, respectively. The subscript *i* = 1 and 2 refer to the outer and inner curved beams, respectively. The widths of the lateral beams and outer curved beams are fixed as 0.9 mm. Therefore, the shape of the outer curved beams is determined by the design variables L_1 and h_1 , and the shape of the inner curved beams is determined by the design variables L_2 , h_2 , and w_2 . The outof-plane thickness of the lateral beams and curved beams is taken as 5.0 mm. Hence, the number of the design variables is 17. Table 1 lists the lower and upper bounds on the design variables. The guide beam and the shuttle have their dimensions indicated in Fig. 3(b). The out-of-plane thickness of the guide beam and the shuttle mass is 6.5 and 5.0 mm, respectively.

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Variables ($i = 2, 3, 4$)	Lower bound (mm)	Upper bound (mm)
x coordinate of B_{ui}	-10	3
y coordinate of B_{ui}	0	20
x coordinate of B_{li}	-10	3
y coordinate of B_{li}	-20	0
L_{1}	20	40
h_1	7.4	9.5
L_2	40	60
h_2	11	16
<i>W</i> ₂	0.8	1.4

Table 1. Lower and upper bounds on the design variables.



Fig. 4 Flowchart of the optimization procedure.

An optimization procedure is developed and outlined in Fig. 4. The nondominated sorting genetic algorithm [18] is applied to the optimization of the shape of lateral and curved beams. The algorithm is suitable for solving constrained multiobjective problems. The efficiency of the nondominated sorting genetic algorithm lies in a way multiple objectives are reduced to a dummy fitness function using nondominated sorting procedure [19]. Let F(X), defined on the design space X, be an objective space, and $X^i \in X$. If a vector $F(X^1)$ is partially less than another vector $F(X^2)$, we say that the solution $F(X^1)$ dominates $F(X^2)$, where no value of $F(X^2)$ is less than $F(X^1)$ and at least one value of $F(X^2)$ is strictly greater than $F(X^1)$. The optimal solutions to a multiobjective minimization problem can be taken as the nondominated solutions. In the optimization process as shown in Fig. 4, initially, the displacement c, the operational range D, constraints on the design variables, number of generations, and size of populations are specified. The objective functions of the optimization problem are

$$Min \quad \int_{\delta=b}^{\delta=c} (f - f_c)^2 d\delta \tag{3}$$

$$Min \quad \left| \frac{f_d}{f_e} - 1 \right| \tag{4}$$

$$Min \quad \left| \frac{f_d}{f_c} - S \right| \tag{5}$$

where f_c , f_d , and f_e are the output forces of the CFBM at the displacements of c, d, and e, respectively, and S is defined as the specified ratio of the output force f_d to f_c . The purpose of Eq. (3) is to minimize the fluctuation of the output force f compared to f_c . The formulation of Eq. (3) is similar to that adopted by Meaders and Mattson [3]. As shown in Fig. 2(a), when the shaded area is smaller, the f- δ curve in the operational range gets flatter and a nearly constant output force can be obtained [3]. The objective function of Eq. (4) is selected for assurance of a high level of bistability of the CFBM, where the sign of the f_d and f_e must be opposite and their absolute values must be as close as possible. The objective function of Eq. (5) is formulated in a way to have the flexibility for specifying the ratio of f_d to f_c . A higher value of S indicates a higher value of f_d is required for snap-through of the CFBM. The value of S is taken as 1.2 in this investigation.

Due to the geometry complexity, large motions and flexible beam behaviors of the CFBM, the output force versus displacement (f- δ) curve of the CFBM may not be calculated analytically. Finite element method can be used to analyze geometrically nonlinear behaviors of the mechanism, and is also suitable to model finite axial strain and transverse shear deformation of stout as well as slender beams of the various beam geometries generated during the optimization process. Finite element analysis by a commercial software ABAQUS [20] is utilized to obtain the f- δ curve. The genetic algorithm optimization procedure used in this investigation is programmed with the commercial software MATLAB 7.0. The genetic algorithm, the design variables and the variable constraints are written in a script file of MATLAB. An ABAQUS text file for the static analysis to obtain the f- δ curve is created by the MATLAB file. The output of the ABAQUS static analysis is saved in a text file. The values of the output forces and the corresponding displacements of the CFBM are obtained from the f- δ curve, and are used to calculate the values of the objective functions for the optimization process. The coordinates of the positions of the lateral and curved beams are also created in the script file of MATLAB.



Fig. 5 A mesh for the finite element model.

Due to symmetry, a half model of the mechanism is considered. Fig. 5 shows a mesh for a two-dimensional finite element model. The finite element model has 204 2-node beam elements. A mesh convergence study is performed to obtain accurate solutions of displacement solutions. The relatively dense mesh is adopted in order to obtain converged solutions of various geometry configurations of the mechanism during the optimization process. A Cartesian coordinate system is also shown in the figure. As shown in Fig. 5, a uniform displacement is applied in the -y direction to the right end of the inner curved beam, and the displacements in the x, y directions and the rotational degree of freedom at the anchors are constrained to represent the clamped boundary conditions in the experiment. The displacement in the x direction and the rotational degree of freedom of the symmetry plane are constrained to represent the symmetry conditions due to the loading conditions and the geometry of the model.

In this investigation, the beams are assumed to be linear elastic materials. A polyoxymethylene (POM) material is used for the CFBM. Five tensile test specimens of the POM material are cut into the form of standard dumbbell-shaped test specimens following the ASTM D638-10 test standard for plastic sheeting. Based on the measured engineering stress - engineering strain curves, the average of the Young's modulus of the POM material is 2.5 GPa. The Poisson's ratio of the POM is taken as 0.25 [21]. The commercial finite element program ABAQUS is employed to perform the computations. 2-node beam element B21H is used for the finite element model. B21H is recommended for geometrically nonlinear analyses when the beam undergoes large motions and is very flexible [20]. B21H can model finite axial strain and transverse shear deformation of stout as well as slender beams of the various beam geometries generated during the optimization process.

2.3 Optimization

In the optimization process, the number of generations, N, is set to be 20, and the population of each generation is taken as 20. The values of the displacement c and the the operational range D are specified as 14 mm and 7 mm, respectively. A static analysis of each population is performed in order to find its f- δ curve. This optimum design of the CFBM using the 17 design variables is obtained by the optimization procedure outlined in Fig. 4. Table 2 lists the values of the design variables of the optimum design. Fig. 6(a) shows the f- δ curve of the optimum design of the CFBM, where b = 7.00 mm, c = 14.00 mm, d = 17.25 mm, g = 23.46 mm, $f_b = 10.74$ N, $f_c = 11.09$ N, and $f_d = 13.19$ N. Therefore, the operational range, displacement between b

and c, is 50% of the displacement between a and c, as specified (see Fig. 6(a)). As seen in the figure, when the displacement of the shuttle mass increases from 0 (the first stable equilibrium position), the output force increases initially, reaches the nearly constant output force value, then reaches its maximum value. As the displacement increases further, in the event of snap-through of the mechanism the output force decreases, attains its minimum value, then increases again to reach 0, the mechanism attains its second stable equilibrium position g. The results indicate that the design of the CFBM has a nearly constant output force in the specified operational range and the bistability for overload protection. In order to avoid yielding of the CFBM under loading, the stress in the mechanism should not exceed the yield strength, 72 MPa, of the POM material used for the CFBM in this investigation.

	0 1
Variable	Values (mm)
(x, y) of the point B_{u2}	(-2.959, 8.447)
(x, y) of the point B_{u3}	(0.838, 10.823)
(x, y) of the point B_{u4}	(-6.418, 13.885)
(x, y) of the point B_{l2}	(-2.980, -7.808)
(x, y) of the point B_{l3}	(-2.547, -14.330)
(x, y) of the point B_{l4}	(-0.389, -15.810)
L_1	27.914
h_1	9.378
L_2	46.832
h ₂	12.139
	0.867

Table 2. The values of the design variables of the optimum design.



Fig. 6 (a) A f- δ curve; (b) strain energy curve based on a finite element model of the optimized solution.

Fig. 6(b) shows the strain energy curves as functions of the displacement of the optimum design. When the displacement of the mechanism increases, the strain energy of the flexible hinge increases, and the strain energy of the inner curved beam increases slightly, then reaches an almost constant value. As the displacement of the mechanism increases further before the mechanism snaps to the second stable state, the majority of the strain energy is absorbed by the flexible hinge, where the CFBM has the characteristics of nearly constant output force over a range of displacement. As the CFBM deflects further, the strain energy in the flexible hinge and inner curved beam

increases. As the displacement of the mechanism increases beyond a certain value, where the CFBM has the maximum output force, the strain energy of the flexible hinge drops suddenly and the strain energy of the inner curved beam increases abruptly. While the compression strain energy in the inner curved beam increases to a maximum; the mechanism snaps towards its second stable equilibrium state. After the snap-through of the CFBM, the strain energy of the flexible hinge is nearly zero and the strain energy of the mechanism attains a local minimum value, corresponding to its second stable equilibrium position.

The simple static analyses of the CFBM are employed in order to obtain the f- δ curve of the mechanism in the design stage. As shown in Fig. 6(a), the f- δ curve of the CFBM is highly nonlinear, and the finite element model is required to model this nonlinear force-displacement relation of the mechanism. The nonlinearities are due to the post-buckling behavior and geometric nonlinearity.



Fig. 7 (a) A photo of a fabricated CFBM. (b) A close-up view of the flexible hinge.

3. Fabrication and testing

In order to prove the design of the CFBM for the constant output force and bistability, prototypes of the mechanisms are fabricated. The prototypes are carved by a milling machine (PNC-3100, Roland DGA Co., Japan) from the POM material.

Dimensions of the prototypes are based on the optimal design parameters obtained from the optimization process. Fig. 7(a) is a photo of a fabricated CFBM. Fig. 7(b) is a close up view of the flexible hinge.



Fig. 8 A photo of the experimental setup.

Fig. 8 is a photo of the experimental setup for measurement of the f- δ curve of the CFBM. The mechanism is mounted on a steel plate. Then, the plate is held vertically by a fixture. A force gauge (FG5020, Lutron Electronic Enterprise Co., Ltd., Taiwan) used to apply a force to the CFBM is held by a micro manipulator. Initially, the mechanism is in its first stable equilibrium position. The probe tip of the force gauge is pushed slowly against the top surface of the shuttle mass of the mechanism. The pressing force applied to the mechanism is increased until it snaps toward its second stable equilibrium position. The displacement of the shuttle mass and the reading of the force gauge are recorded. With the force gauge attached to the bottom surface of the shuttle mass and pulled by the micro manipulator through a metal ring, a f- δ curve for backward motion of the CFBM can be obtained. A CCD camera is used for capturing the successive images of the motion of the mechanism.



Fig. 9 Snapshots for forward motion (a-c) and backward motion (d-f).

4. Results and discussions

Using the experimental setup, the constant force output and bistable behavior of the CFBM are demonstrated. The experimental f- δ curves of the forward and backward motion of the optimum design of the mechanism are also obtained. Fig. 9(a-c) and 9(d-f) show sequences of snapshots from experiments for forward and backward motion, respectively. As shown in Fig. 9(a-c), a force is applied on the top surface of the shuttle mass for forward motion. When the magnitude of the force is increased, the shuttle mass moves downward. As the force reaches a certain maximum value, the probe tip loses contact with the top surface of the shuttle mass, and the mechanism snaps into its second stable equilibrium position. For backward motion, the shuttle mass is pulled upward by a

ring attached to the probe tip of the force gauge, and, therefore, a force is applied on the bottom surface of the shuttle mass (see Fig. 9(d)). As the magnitude of the pulling force reaches a certain maximum value, the ring on the probe tip loses contact with the bottom surface of the shuttle mass, and the mechanism snaps into its first stable equilibrium position (see Fig. 9(e,f)).



Fig. 10 f- δ curves of the fabricated CFBM for (a) forward, (b) backward motion.

Fig. 10(a) and (b) show the f- δ curves of the mechanism bases on experiments and finite element computations for forward and backward motion, respectively. In order to model the force applied to the shuttle mass through the probe tip of the force gauge in the experiments (see Fig. 8), a contact is defined between a rigid body, through which a force is applied, and a node of a beam element of the shuttle mass in the finite element analyses. As shown in Fig. 10(a), a nearly constant force is observed in the operational range from the displacement of 7.00 to 14.00 mm based on both the experiments and finite element analyses. For forward motion, before the applied force reaches the maximum value, the experimental results are slightly higher than those based on the finite element analyses. As the probe tip pushes the shuttle mass further, a reduction in the output force indicates the incoming of the snap-through of the CFBM (see Fig. 6). In the event of snap-through, the probe tip loses contact with the shuttle mass and the mechanism jumps to its second stable equilibrium position. For backward motion, before the applied force reaches the maximum magnitude value, the experimental results are slightly lower than those based on the finite element analyses. As the probe tip pushes the shuttle mass further, a reduction in the magnitude of the output force indicates the incoming of the snap-through of the CFBM. In the event of snap-through, the probe tip loses contact with the shuttle mass and the mechanism jumps to its first stable equilibrium position.

It is observed that, before the snap-through, the magnitudes of the output force based on the experiments are greater than those of the finite element analyses. This can be attributed to the uncertainties in geometry and loading conditions of the experiments. The fabricated mechanism has slightly larger beam widths than the designed value due to the manufacturing error in the milling process. The contact of the probe tip of the force gauge with the surface of the shuttle mass is not fixed, where sliding may occur, and the alignment of the force gauge with the symmetry plane of the CFBM may not be perfect during the experiments, where twisting of the CFBM may happen. Also, the effects of overlapping beam widths at the connecting nodes are not considered by the beam element B21H employed in ABAQUS. As a result, the effective flexural rigidity of each beam near the connecting nodes is underestimated in the finite element analyses.

In this investigation, the value of the constant output force is not a design variable. It can be specified as a design constraint of the optimization procedure at the expense of computational time. The average computational time on a personal computer with the Intel Core 2 DuoTM processor at 2.53 and 1.67 GHz and 3.50 GB RAM for the design case considered is around 80 hours. The preload region, the displacement range from *a* to *b* (see Fig. 2), should be as small as possible in order to increase the operational range. However, the sharp change in structure stiffness may cause divergence in the finite element analyses due to the high nonlinearity of the structural response.

5. Conclusions

A CFBM has been successfully designed by an optimization procedure and validated by experiments. Using the design methodology, a specified constant force range can be attained without exceeding the yield stress of the material. The CFBM consists of beams with profiles of cubic Bézier curves and cosine curves. The combination of a chevron-type mechanism and a bistable mechanism of the CFBM provides the constant output force for force regulation and the bistability for impact/overloading protection. The feasibility of using the mechanism to achieve constant output force and bistability is confirmed by finite element analyses. In order to confirm the effectiveness of the CFBM, prototypes of the mechanism are fabricated using a simple milling process. The force versus displacement curve of the mechanism exhibits

a constant output force of nearly 11 N in the specified operational range of 7.00 to 14.00 mm and a maximum output force of nearly 13 N for the bistable snap-through of the mechanism. The presented design method of the CFBM provides a simple and efficient means of attaining a constant output force in a specified operational range and overload protection of the mechanism under a specified maximum load. The proposed CFBM may be incorporated into an ultrasonic machine tool against damage caused by excessive contact force between the tool and unknown surface conditions while maintaining a constant force output in the operational range of the system.

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- Fig. 2 (a) A typical force versus displacement curve of the CFBM and the corresponding positions at displacement a (b), displacement c (c), displacement d (d), and displacement g (e).
- Fig. 3 (a) A schematic of the upper-left branch of the CFBM. (b) Dimensions of the guide beam and the shuttle mass.
- Fig. 4 Flowchart of the optimization procedure.
- Fig. 5 A mesh for the finite element model.
- Fig. 6 (a) A f- δ curve and maximum stress versus displacement curve; (b) strain energy curve based on a finite element model of the optimized solution.
- Fig. 7 (a) A photo of a fabricated CFBM. (b) A close-up view of the flexible hinge.
- Fig. 8 A photo of the experimental setup.
- Fig. 9 Snapshots for forward motion (a-c) and backward motion (d-f).
- Fig. 10 f- δ curves of the fabricated CFBM for (a) forward, (b) backward motion.