A quadristable compliant mechanism with a bistable structure embedded in a surrounding beam structure

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Abstract

A quadristable mechanism with a bistable structure embedded in a surrounding beam structure is developed. Three stable equilibrium positions are within the range of the forward motion of the mechanism, and the fourth stable equilibrium position can only be reached on the backward motion. The quadristability of the mechanism originates from combined compression and bending of the beam structures. Finite element analyses are used to characterize the quadristable behavior of the mechanism under static loading. A design formulation is proposed to find the shape of beams of the mechanism. Prototypes of the mechanism are fabricated and tested. The characteristics of the mechanism predicted by theory are verified by experiments. The design example presented in this investigation demonstrates the effectiveness of the optimization approach for the design of the quadristable compliant mechanism. The proposed mechanism has no movable joints and gains its mobility from the deflection of flexible members. This compliant mechanism can be easily miniaturized, offering a significant advantage for application in micro actuators, micro sensors and microelectromechanical systems. Keywords: Quadristable mechanism; bistable; optimization

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1. Introduction

Multistable mechanisms, which provide multiple stable equilibrium positions within their operation range, can be used to design systems with both power efficiency and kinematic versatility, oftentimes two conflicting goals. With the concept of multistable mechanisms, a wide range of operating regimes or novel mechanical systems without undue power consumption can be created [1]. Substantial interest has focused on design of bistable [2-9] and tristable mechanisms [10-15].

Few mechanisms with four or more mechanically stable positions have been reported. Han et al. [16] developed a planar quadristable mechanism (QM) using two pairs of curved beams to achieve quadristability with two stable positions in each of two orthogonal directions. The sequence of switching between stable positions can be altered by selectively actuating the mechanism in one of the two orthogonal directions. Oh and Kota [17] synthesized a QM with four stable rotational orientations. Their design is based on a combination of two bistable rotational mechanisms. A generalized scheme for designing combinations of bistable mechanisms in series is still needed. King et al. [1] proposed a QM consisting of a rotating compliant beam with an armature magnet attached to it and an array of stator magnets. Fields of strain energy, gravitational energy and magnetic energy are all involved in the stability modes of their mechanism, and the inherently nonlinear nature of the energy storage elements may require a significant effort to obtain a feasible design. Hafez et al. [18] proposed a robotic device with a large number of degrees of freedom, which can be taken as a large number of stable positions/orientations. The discrete nature of their mechanism is ensured by the use of bistable mechanisms. With complexity of the assembly of modular parallel platforms and a network of flexible and rigid members with embedded actuators, their mechanism can be used for tasks which require a robot to operate in a three-dimensional space, such as camera placement and light positioning.

Multiple passive stable equilibrium configurations enable the function of the QMs to be more versatile, while the actuators and control stay simple [1]. For example, QMs can be used for multiple switching and optical networking [16]. Fig. 1 shows a 'ball-on-the-hill' analogy for a QM, similar to a figure presented by Chen et al. [14]. The elevation of the ball is related to the potential energy of the ball due to the gravity. A ball should be stable if located in the valleys, marked as A, C, E and G, and unstable if placed on the hills, marked as B, D, F, and H. The ball located at the deepest valley A has the global minimum of the potential. The valleys C, E and G are local minima of the potential, which can be taken as metastable states where the ball is in equilibrium but is susceptible to fall into lower-energy states with finite disturbance. The ball located on a hill, an unstable position, would slide to a nearby valley under a small perturbation, and stay in the valley, a stable equilibrium position, under a small disturbance. As the ball goes from A to E, through B, C, and D, and returns to A through F, G and H, it encounters four stable equilibrium positions, A, C, E, and G. The strain energy of a QM

has four stable equilibrium positions which require no power input to maintain the state under small disturbances.

This paper describes a design of a compliant QM. The proposed QM has a curved-beam bistable mechanism embedded in a curved-beam structure. Multi-stability is provided by buckling of curved-beam structures of the mechanism. The design concept of combining two bistable mechanisms has been reported by Han et al. [16] and Oh and Kota [17], where the quadritability originates from bistable behaviors of the mechanism along two orthogonal directions, two bistable positions in each direction, [16] or of a combined motion of two bistable rotational mechanisms [17]. The motion of the proposed QM is translational in a one-dimensional manner. An optimization method is used to design the QM. Finite element analyses are carried out to evaluate the mechanical behaviors of the design obtained by the optimization procedure. Prototypes of the device are fabricated using a milling process. Experiments are carried out to demonstrate the effectiveness of the QM.

Geometry nonlinearities due to large deflections are commonly encountered in compliant mechanisms [21]. Large strain of the QM causes significant changes in its geometry. Modeling of force-deflection characteristics of multistable compliant mechanisms can be performed by the pseudo-rigid-body model (PRBM) [14]. However, in order to accurately describe the behavior of compliant mechanisms using PRBM, where to place the added springs and what value to assign their spring constants are important. Theory of static Euler buckling of a double-clamped slender beam can be used to model the force-deflection and snap-through behavior of the compliant beams [22]. This classical treatment has been used for the analysis of compliant bistable mechanisms with curved double-clamped beams with fixed ends [22-25]. The proposed QM is composed of double-clamped beams with fixed ends, where the embedded bistable mechanism has sliding clamped boundary conditions. The use of the Euler's beam buckling theory for the QM needs further investigation. In this paper, finite element models of the QM applying Euler's beam buckling theory are coupled to an optimization method for design of a QM.

2. Design

2.1 Operational principle

A schematic of the QM is shown in Fig. 2(a). A Cartesian coordinate system is also shown in the figure. The mechanism consists of a shuttle mass, a guide beam, inner curved beams and outer curved beams. The inner curved beams clamped at one end by the shuttle mass and fixed at the other end by the guide beam acts similar to a bistable mechanism of curved beam type. The outer curved beams with one end clamped at the guide beam and the other end fixed at the anchor also behave similar to a bistable mechanism of curved beam type. The shuttle mass and the guide beam are employed to prevent the mechanism from twisting during operation, and are designed to be stiff. Curved beams with large thicknesses could be used to prevent twisting of the mechanism. Due to the constraint of machining depth of the available milling machine, the guide beam is employed instead. Upon the application of a force *F* to the shuttle mass in the -y direction, the forward direction, the outer curved beams deflect initially, releasing the strain energy. The compression energy in the outer curved beams increases to a maximum at a certain displacement of the mechanism, but then decreases; the mechanism snaps towards its second stable position, as shown in Fig. 2(b). As the QM deflects further, the bending energy in the outer and inner curved beams increases. While the compression energy in the inner curved beams increases to a maximum at a certain displacement of the mechanism, but then decreases; the mechanism snaps towards its third stable position, as shown in Fig. 2(c). Next, with the force applied in the + y direction, the backward direction, the outer curved beams deflect initially, releasing the strain energy. As the mechanism deflects further, the compression energy in the outer curved beams increases to a maximum and then decreases; the mechanism snaps towards its fourth stable position, as shown in Fig. 2(d). The four stable positions shown in Figs. 2(a-d) are correspondent to the four valleys, A, C, E, and G, illustrated in Fig. 1, respectively.

The analytical background for snap-through of the QM can be referred to the work of Vango [20]. Vangbo [20] treated the snap-through behavior of a double-clamped curved beam using Euler's beam buckling theory [24]. Considering both the effects of bending and compression and using an energy approach, Vangbo [20] derived a parametric form of the displacement as a function of the applied force of the beam. Evaluating the bending and compression energy terms of his analytical solution, it is found that bending energy is larger than compression energy when the beam is loaded initially; as the displacement of the beam increases, compression energy increases rapidly and bending energy decreases; after the event of snap-through of the beam, bending energy starts to increase again while the compression energy remains constant due to a constant stress normal to the cross-section of the beam.

Fig. 3 shows a typical reaction force versus displacement (f- δ) curve of the QM. The configurations of the QM in its four stable positions are shown in the inlets. A Cartesian coordinate system is also shown in the figure. When a force is applied to the mechanism through the shuttle mass, the value of the reaction force of the QM increases initially, and reaches a local maximum, $F_{1_{max}}$. When the force applied to the mechanism is greater than $F_{1_{max}}$, the outer beams of the QM buckle and the reaction force decreases, reaches a local minimum, $F_{1_{min}}$, then increases and attains a value of 0, where the QM is in its second stable position b. As the shuttle mass is displaced further, the reaction force increases, reaches a local maximum, $F_{2_{max}}$. When the force applied to the mechanism is greater than $F_{2_{max}}$, the inner beams of the QM buckle and the reaction force decreases, reaches a local minimum, $F_{2_{max}}$. When the force applied to the mechanism is greater than $F_{2_{max}}$, the inner beams of the QM buckle and the reaction force decreases, reaches a local minimum, $F_{2_{max}}$, then increases again and attains a value of 0, where the QM is in its third stable position c.

When the direction of the applied force is reversed, the value of the reaction force decreases initially, a negative value means the reaction force is in the -y direction, and reaches a local minimum. As the shuttle mass is displaced further in the -y direction, the outer beams buckle and the value of the reaction force increases, and reaches a local maximum, then the reaction force decreases, and attains a value of 0, where the QM is in its fourth stable position d. As the shuttle mass is displaced further, the value of the reaction force decreases, reaches a local minimum, then the inner beams buckle and the increase of the displacement of the shuttle mass, the reaction force reaches a local maximum, then decreases and attains a value of 0, where the QM returns to its first stable position a.

2.2 Design

The design of the QM is based on an optimization procedure where the shape of the outer and inner curved beams is optimized via the parameters of cosine curves. Due to symmetry, only a quarter model of the mechanism is considered. Fig. 4(a) is a schematic of the quarter model. The shape of the curved beams is

$$y_r = \frac{h_i}{2} \left(1 - \cos \frac{\pi x_r}{L_i} \right) \tag{1}$$

where (x_r, y_r) is the position vector with the reference origin at the left end of the curved beams. *L* and *h* are the span and apex height of the curved beams, respectively. The subscript *i* = 1 and 2 refer to the outer and inner curved beams, respectively. The widths of the outer and inner curved beams are indicated in Fig. 4(a) as w_1 and w_2 , respectively. Therefore, the shape of the outer curved beams is determined by the design variables L_1 , h_1 and w_1 , and the shape of the inner curved beams is determined by the design variables L_2 , h_2 , and w_2 . The thickness of the outer and inner curved beams is taken as 5 mm. Hence, the number of the design variables is 6. Table 1 lists the lower and upper bounds on the design variables. The guide beam and the shuttle have their dimensions indicated in Fig. 4(b). The thickness of the guide beam and the shuttle mass is 6.5 and 5 mm, respectively.

An optimization procedure is developed and outlined in Fig. 5. The nondominated sorting genetic algorithm [25] is applied to the optimization of the shape of outer and inner curved beams. The algorithm is suitable for solving constrained multiobjective problems. In the optimization process as shown in Fig. 5, initially, constraints on the design variables, number of generations, and number of populations are specified. The objective functions of the optimization problem are

$$Min \quad \left| \frac{F_{1_{\min}}}{F_{2_{\min}}} - 0.5 \right| \tag{2}$$

$$Min \quad \begin{vmatrix} F_{1_{\text{max}}} \\ -F_{1_{\text{min}}} \\ -1 \end{vmatrix}$$
(3)

$$Min \quad \left| \frac{F_{2_{\max}}}{-F_{2_{\min}}} - 1 \right| \tag{4}$$

where $F_{l_{max}}$, $F_{l_{min}}$, $F_{2_{max}}$, and $F_{2_{min}}$ are the reaction forces of the QM indicated in Fig. 3. In order to obtain a design of the QM with quadristability, the objective function of Eq. (2) aims to ensure that the outer curved beams buckle before the inner curved beams, otherwise the mechanism might exhibit tristability other than quadristability. The objective function of Eq. (3) is selected for assurance of a high level of snap-through behavior of the QM so that the mechanism can settle down to its second stable position easily. The objective function of Eq. (4) is also formulated for assurance of a high level of snap-through behavior of the QM to have the mechanism settle down to its third stable position and to eliminate the possibility of returning to its second stable position under influence of small disturbance.

Due to the geometry complexity, the reaction force versus displacement (f- δ) curve of the QM cannot be calculated analytically. Finite element analysis by a commercial software ABAQUS [26] is utilized to obtain the f- δ curve. The genetic algorithm optimization procedure used in this investigation is programmed with the commercial software MATLAB 7.0. The genetic algorithm, the design variables and the variable constraints are written in a script file of MATLAB. An ABAQUS text file for

the static analysis to obtain the f- δ curve is created by the MATLAB file. The output of the ABAQUS static analysis is saved in a text file. The values of the reaction forces and the corresponding displacements of the QM are obtained from the f- δ curve, and are used to calculate the values of the objective functions for the optimization process. The coordinates of the positions of the outer and inner curved beams are also created in the script file of MATLAB.

Fig. 6 shows a mesh for a two-dimensional finite element model. The finite element model has 124 2-node beam elements. The width and thickness of the beam element B21 employed in the finite element analyses are specified according to the dimensions of the QM. A mesh convergence study is performed to obtain accurate solutions of displacement solutions. A Cartesian coordinate system is also shown in the figure. As shown in Fig. 6, a uniform displacement is applied in the -y direction to the right end of the inner curved beam, and the displacements in the x, y directions and the rotational degree of freedom at the anchors are constrained to represent the clamped boundary conditions in the experiment. The displacement in the x direction and the rotational degree of freedom of the symmetry plane are constrained to represent the symmetry conditions due to the loading conditions and the geometry of the model.

In this investigation, the beams are assumed to be linear elastic materials. A polyoxymethylene (POM) material is used for the QM. The Young's modulus and Poisson's ratio of the POM are 3.1 GPa and 0.25, respectively. As described above, the commercial finite element program ABAQUS is employed to perform the computations. 2-node beam element B21 is used for the finite element model.

2.3 Optimization

In the optimization process, the number of generations is set to be 50, and the population of each generation is taken as 20. A static finite element analysis of each population is performed in order to find its f- δ curve. The optimum design of the QM using the 6 design variables is obtained by the optimization procedure outlined in Fig. 5. Table 2 lists the values of the design variables of the optimum design. Fig. 7 shows the distribution of the population of the 10th and 50th generations in the optimization process. The x, y, and z -coordinates represent the values of the objective functions $F_{1_{\min}}/F_{2_{\min}} = 0.5$, $F_{1_{\max}}/(-F_{1_{\min}}) = 1$, and $F_{2_{\max}}/(-F_{2_{\min}}) = 1.$, respectively. Using the genetic algorithm, the optimum values of $F_{1_{\min}}/F_{2_{\min}} = 0.5$, $F_{1_{\max}}/(-F_{1_{\min}}) = 1$ and $F_{2_{\max}}/(-F_{2_{\min}}) = 1.$ are 0.147, 1.820, and 2.956, respectively, after 50 generations. As shown in the figure, the values of the objective functions of the optimal design are decreased dramatically compared to their values in the 10th generation.

Fig. 8(a) shows the f- δ curve of the optimum design of the QM when the shuttle mass is displaced in the forward direction, -y direction, where a = 0 mm, b = 10.26 mm, c = 20.05 mm, $F_{1_{max}} = 5.59$ N, $F_{1_{min}} = -1.98$ N, $F_{2_{max}} = 12.09$ N, and $F_{2_{min}} = -3.06$ N. As seen in the figure, when the displacement of the shuttle mass increases from 0 (the first stable equilibrium position), the reaction force increases initially, then reaches a local maximum value, $F_{1_{max}}$. As the displacement increases further, the reaction force decreases; in the event of snap-through of the outer beams of the mechanism, where the strain energy of the outer beams reaches a local maximum $U_{1_{max}}$ shown in Fig. 8(b), the reaction force reaches a value of 0, then decreases and attains a local minimum, $F_{1_{min}}$.

With the increasing displacement of the shuttle mass, the reaction force increases again and reaches a value of 0, where the mechanism is in its second stable equilibrium position b. As the shuttle mass is displaced further, the reaction force increases, reaches a local maximum value, $F_{2_{max}}$. As the displacement increases further, the reaction force decreases and reaches a value of 0; then the inner beams of the QM buckle, where the strain energy of the inner beams reaches a local maximum $U_{2_{max}}$ shown in Fig. 8(b), and the reaction force decreases, attains a local minimum, $F_{2_{min}}$. Next the reaction force increases again, reaches a value of 0, and the mechanism reaches its third stable equilibrium position c. Fig. 8(a) also shows the von Mises stress as a function of the displacement based on the finite element computations. The highest stress, 54 MPa, occurs in the event of the second snap-through of the QM. In order to avoid yielding of the QM under loading, the stress in the mechanism should not exceed the yield strength, 72 MPa, of the POM material used for the QM in this investigation. As seen in the figure, the value of the highest stress is less than the yield strength of the POM material.

Fig. 8(b) shows the strain energy curves as functions of the displacement of the optimum design. When the displacement of the mechanism increases, the strain energy of the outer beams increases, and the strain energy of the inner beams increases slightly. As the displacement of the mechanism increases further before the mechanism snaps to the second stable equilibrium position, b, the majority of the strain energy is absorbed by the outer beams. In the event of snap-through of the outer beams of the mechanism, the strain energy reaches a local maximum $U_{1_{max}}$, then decreases and attains a local minimum shown in Fig. 8(b). As the QM deflects further, the strain energy in the outer and inner curved beam increases. As the displacement of the mechanism increases beyond a

certain displacement, where the inner beams of the QM buckle, the strain energy of the inner beams drops abruptly from a local maximum, $U_{2_{max}}$, and the strain energy of the outer beams decreases gradually. While the strain energy of the QM decreases to a local minimum; the mechanism settles in its third stable equilibrium position, c.

Fig. 8(c) shows the f- δ curve of the optimum design of the QM when the shuttle mass is displaced in the backward direction, +y direction, where d = 12.44 mm with the origin located in the third stable equilibrium position, c. The reaction force is positive if it points against the direction of the displacement of the shuttle mass. In Fig. 8(c), the positive direction of the coordinate axis for the reaction force is directed downward. When the displacement of the shuttle mass increases from 0, the reaction force increases initially, then reaches a local maximum value, $F_{3_{max}}$. As the displacement increases further, the reaction force decreases; in the event of snap-through of the outer beams of the mechanism, where the strain energy of the outer beams reaches a local maximum $U_{3_{max}}$ shown in Fig. 8(d), the reaction force reaches a value of 0, then decreases and attains a local minimum, $F_{3_{min}}$. With the increasing displacement of the shuttle mass, the reaction force increases again and reaches a value of 0, where the mechanism is in its fourth stable equilibrium position d. As the shuttle mass is displaced further, the reaction force increases, reaches a local maximum value, $F_{\rm 4_{max}}$. As the displacement increases further, the reaction force decreases and reaches a value of 0; then the inner beams of the QM buckle, where the strain energy of the inner beams reaches a local maximum $U_{4_{max}}$ shown in Fig. 8(d), and the reaction force decreases, attains a local minimum, $F_{4_{\min}}$. Next the reaction force increases again, reaches a value of 0, and the

mechanism returns to its first stable equilibrium position a. Fig. 8(c) also shows the von Mises stress as a function of the displacement based on the finite element computations. The highest stress, 56 MPa, which is less than the yield strength of the POM material, occurs in the event of the second snap-through of the QM when the shuttle mass is displaced in the backward direction.

Fig. 8(d) shows the corresponding strain energy curves as functions of the displacement in the backward direction. When the displacement of the mechanism increases, the strain energy of the outer and inner beams increases. In the event of snap-through of the mechanism towards its fourth stable equilibrium position, d, the strain energy of the outer beams reaches a local maximum $U_{3_{max}}$, then decreases as shown in Fig. 8(d). As the QM deflects further, the strain energy in the inner beams increases initially and then decreases, and the strain energy in the outer beams decreases. As the displacement of the mechanism increases beyond a certain displacement, where the inner beams of the QM buckle, the strain energy of the inner beams drops abruptly from a local maximum, $U_{4_{max}}$. While the strain energy of the QM decreases to a local minimum; the mechanism returns to its first stable equilibrium position, a.

The simple static analyses of the QM are employed in order to obtain the f- δ curve of the mechanism in the design stage. As shown in Fig. 8(a), the f- δ curve of the QM is highly nonlinear, and the finite element model is required to model this nonlinear force-displacement relation of the mechanism. The nonlinearities can be attributed to the post-buckling behavior, geometric nonlinearity and damping effects.

3. Fabrication and testing

In order to prove the quadristability of the QM design, prototypes of the mechanisms are fabricated. The prototypes are carved by a milling machine (PNC-3100, Roland DGA Co., Japan) from the POM material. The thickness of the POM material is 6.5 mm. The cutting tool of the milling machine has a maximum machining depth of 7.0 mm. Dimensions of the prototypes are based on the optimum design of the optimization process. Fig. 9 is a photo of a fabricated QM.

Fig. 10 is a photo of the experimental setup for measurement of the f- δ curve of the QM. The mechanism is mounted on a steel plate. Then, the plate is held vertically by a fixture. A force gauge (FG5020, Lutron Electronic Enterprise Co., Ltd., Taiwan) used to apply a force to the QM is held by a micro manipulator. The displacement of the shuttle mass of the QM is taken as the displacement of a slider of the micro manipulator. For the loading ranging from -4 N to 12 N, the displacement of the probe tip of the force gauge relative to the slider is less than 200 μ m. For the purpose of proving the design concept of the QM, this measurement error is negligible compared to the displacement range of the QM, 0 mm to 20 mm. Initially, the mechanism is in its first stable equilibrium position. The probe tip of the force gauge is pushed slowly against the top surface of the shuttle mass of the mechanism for forward motion of the QM. While affixing a metal ring to the probe tip and attaching the metal ring to the bottom surface of the shuttle mass, the shuttle mass can be pulled by the micro manipulator and a f- δ curve for backward motion of the QM can be obtained. The displacement of the shuttle mass and the reading of the force gauge are recorded. A CCD camera is used for capturing successive images of the motion of the mechanism.

4. Results and discussions

Using the experimental setup, the quadristable behavior of the QM is demonstrated. The experimental f- δ curves of the forward and backward motion of the optimum design of the mechanism are also obtained. Fig. 11(a-c) and 11(d-f) show sequences of snapshots from experiments for forward and backward motion, respectively. As shown in Fig. 11(a-c), a force is applied on the top surface of the shuttle mass for forward motion. When the magnitude of the force is increased, the shuttle mass moves forward. As the force reaches a certain maximum value, the probe tip loses contact with the top surface of the shuttle mass, and the mechanism snaps into its second stable position (Fig. 11(b)). Then the probe tip is moved to contact with the top surface of the shuttle mass again and pushed slowly until the QM snaps into its third stable position (Fig. 11(c)). For backward motion, the shuttle mass is pulled backward by a ring affixed to the probe tip of the force gauge, and a force is applied on the bottom surface of the shuttle mass (see Fig. 11(d)). As the magnitude of the pulling force reaches a certain maximum value, the ring on the probe tip loses contact with the bottom surface of the shuttle mass, and the mechanism snaps into its fourth stable position (see Fig. 11(e)). Then the ring affixed to the probe tip is moved to contact with the bottom surface of the shuttle mass again and pushed slowly until the QM snaps into its first stable position (see Fig. 11(f)).

Fig. 12(a) and (b) show the f- δ curves of the mechanism bases on experiments and finite element computations for forward and backward motion, respectively. As shown in Fig. 12(a) for forward motion, before the reaction force reaches a value of 0, where the snap-through of the outer beams of the QM occurs, the experimental results are in good agreement with those based on the finite element analyses. In the event of snapthrough, the probe tip loses contact with the shuttle mass, and the QM snaps into its second stable position as seen by the discontinuous curve of the experiments. Α displacement controlled approach is adopted in the finite element analyses, where the snap-through behavior is signaled by the 0 value of the reaction force, and the negative values of the reaction force are obtained while the shuttle mass is displaced further towards the second stable position of the QM based on the finite element analyses. As the shuttle mass is displaced further, a local maximum of the reaction force is reached, then a reduction in the reaction force indicates the second snap-through of the QM. At the second snap-through, the probe tip loses contact with the shuttle mass and the mechanism jumps to its third stable position. As shown in Fig. 12(b), the experimental results of the f- δ curve of the backward motion are also in good agreement with the finite element analyses. The reaction force is positive if it points against the direction of the displacement of the shuttle mass. In Fig. 12(b), the positive direction of the coordinate axis for the reaction force is directed downward. An abrupt increase in the reaction force is observed at the second snap-through of the backward motion based on the finite element computations, where the strain energy of the inner beams of the QM drops suddenly to a value of nearly zero, the strain energy of the outer beams increases abruptly, and the total strain energy of the QM decreases, as seen in Fig. 8(d)

The discrepancies between the experiments and finite element analyses can be attributed to uncertainties in material properties, geometry and loading conditions of the experiments. Measurement errors of force and displacement also contribute to the discrepancies. The fabricated mechanism has slightly larger beam widths than the designed value due to manufacturing error in the milling process. The contact of the probe tip of the force gauge with the surface of the shuttle mass is not fixed, where sliding may occur, and the alignment of the force gauge with the symmetry plane of the QM may not be perfect during the experiments, where twisting of the QM may happen. Also, the effects of overlapping beam widths at the connecting nodes are not considered by the beam element B21 employed in ABAQUS. As a result, the effective flexural rigidity of each beam near the connecting nodes is underestimated in the finite element analyses. The difference between the actual boundary conditions and the modeled boundary conditions in the finite element analyses may also cause discrepancies. The assumption that the beams are connected to rigid segments, which it taken as clamped boundary conditions in the analyses, is reasonable, but not completely accurate and can account for some of the discrepancies.

The prototype QM is indeed a series combination of two bistable mechanisms, where a bistable structure, BS1, is embedded in another bistable structure, BS2, as indicated in Fig. 2(a). Fig. 13(a) and (b) are the schematics of BS1 and BS2, respectively. Quadristability is provided by combining the two bistable equilibrium systems. Fig. 13(c) shows the f- δ curves of BS1 and BS2 based on their individual finite element models. Since the behaviors of the bistable mechanisms, BS1 and BS2, are highly nonlinear, combining such nonlinearities to capture the behavior of the QM can be quite challenging [17]. Oh and Kota [17] presented a mathematical approach for synthesizing multistable compliant mechanisms by combining bistable mechanisms. Using a procedure for generating a potential energy contour plot from potential energy functions of the

individual bistable mechanisms, then obtaining the f- δ curve of the combined mechanism, they identified various types of multistability based on the actuation loads of the individual bistable mechanisms. The actuation loads of the forward and backward motion of BS1, F_{1_f} and F_{1_b} , respectively, and of BS2, F_{2_f} and F_{2_b} , respectively, are illustrated in Fig. 13(c). Since $F_{2_f} < F_{1_f}$ and $F_{2_b} < F_{1_b}$, the QM is identified to have four stable equilibria based on the work of Oh and Kota [17].

As shown in Fig. 4(b), the proposed mechanism has no movable joints, such as revolute or sliding joints, and can be considered as a compliant mechanism [19]. The mechanism gains its mobility from the deflection of flexible members. The monolithic nature of the mechanism allows its fabrication in the microscale, which offers a possibility of its application in microelectromechanical systems. In order to demonstrate the scalability of the design, a finite element analysis of a microscale version of the compliant mechanism is carried out. A common material used in microfabrication, nickel, is selected for the material of the microscale QM. The thickness of the outer curved beams, inner curved beams, guide beam and shuttle mass is taken as 20 μ m. The optimum design of the QM is obtained by the optimization procedure. The Young's modulus and Poisson's ratio of the nickel are 205 GPa and 0.31, respectively [27]. Table 3 lists the values of the design variables of the optimum design. Fig. 14(a) and (b) show the f- δ curve and maximum stress versus displacement curve of the microscale version of the QM for the forward and backward motion, respectively. As shown in the figures, the microscale version of the QM is quadristable, where a = 0 mm, $b = 104.50 \text{ }\mu\text{m}$, c =192.90 µm, d = 118.50 µm, $F_{1_{\text{max}}} = 353.80$ µN, $F_{1_{\text{min}}} = -137.30$ µN, $F_{2_{\text{max}}} = 562.20$

 μ N, $F_{2_{min}} = -190.30 \mu$ N, $F_{3_{max}} = 176.90 \mu$ N, $F_{3_{min}} = -340.40 \mu$ N, $F_{4_{max}} = 87.38 \mu$ N, and $F_{4_{min}} = -347.40 \mu$ N. Fig. 14(a) and (b) also shows the von Mises stress as a function of the displacement based on the finite element computations. The highest stresses for forward and backward motion are 499 MPa and 517 MPa, respectively. These values of the highest stress for the forward and backward motion are less than the yield strength of the nickel, 700 MPa. Scaling down of the design of the QM does not lead to excessive stress concentrations.

The proposed QM provides quadristability in a linear, sequential manner with three stable positions on the forward motion, and the fourth stable position on the backward motion. This characteristic imposes some challenges for its applications, such as multiple switching and optical networking. A potential application of the QM could be a microrobot system on chip for control of droplet dispensing in microchannels in the field of chemical engineering and bioengineering which requires a sequential operation with multistability for efficient reaction and productivity [28]. Another possible application is the control of cascaded rubber-seal valves for gas regulation in microchannels, where rectifying of gas flow can be achieved through the deflection of valve diaphragms [29].

5. Conclusions

A QM has been successfully designed by an optimization procedure and validated by experiments. Using the design methodology, the quadristability of the mechanism can be attained without exceeding the yield strength of the material. The QM consists of beams with profiles of cosine curves. The combination of two bistable mechanisms provides the quadristability of the QM. The feasibility of using the mechanism to achieve quadristability is established by finite element analyses. In order to confirm the effectiveness of the QM, prototypes of the mechanism are fabricated using a simple milling process. The observed force versus displacement curve of the mechanism exhibits a well-defined quadristability. The presented design method of the QM provides a simple and efficient means of attaining a quadristable, planar mechanism with its motion in a linear manner.

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References

- C. King, J.J. Beaman, S.V. Sreenivasan, M. Campbell, Multistable equilibrium system design methodology and demonstration, Journal of Mechanical Design 126 (2004) 1036-1046.
- [2] N.D. Masters, L.L. Howell, A self-retracting fully compliant bistable micromechanism, Journal of Microelectromechanical Systems 12 (2003) 273-280.
- [3] M. Freudenreich, U. Mescheder, G. Somogyi, Simulation and realization of a novel micromechanical bi-stable switch, Sensors and Actuators A 114 (2004) 451-459.
- [4] R. Receveur, C.R. Marxer, R. Woering, V. Larik, and N.-F. de Rooij, Lateral moving bistable MEMS DC switch for biomedical applications, Journal of Microelectromechanical Systems 14 (2005) 1089-1098.
- [5] T. Gomm, L.L. Howell, R.H. Selfridge, In-plane linear displacement bistable microrelay, Journal of Micromechanics and Microengineering 12 (2005) 257-264.
- [6] J. Qiu, J.H. Lang, A.H. Slocum, A bulk-micromachined bistable relay with Ushaped thermal actuators, Journal of Microelectromechanical Systems 14 (2005) 1099-1109.
- [7] B.J. Hansen, C.J. Carron, B.D. Jensen, A.R. Hawkins, S.M. Schultz, Plastic latching accelerometer based on bistable compliant mechanisms, Smart Mater. Struct. 16 (2007) 1967-1972.
- [8] Y.-Y. Yang, B.-T. Liao, W.-C. Kuo, A novel 2 X 2 MEMS optical switch using the split cross-bar design, Journal of Micromechanics and Microengineering 17 (2007) 875-882.
- [9] D.-A. Wang, H.-T. Pham, Y.-H. Hsieh, Dynamical switching of an

electromagnetically driven compliant bistable mechanism, Sensors and Actuators A 149 (2009) 143-151.

- [10] M. Ohsaki, S. Nishiwaki, Shape design of pin-jointed multistable compliant mechanisms using snapthrough behavior, Structural and Multidisciplinary Optimization 30 (2005) 327-334.
- [11] H.-J. Su, J.M. McCarthy, Synthesis of compliant mechanisms with specified equilibrium positions, Proceedings of 2005 ASME Design Engineering Technical Conferences September 24-28, 2005, Long Beach, California, USA, Paper no. DETC2005-85085 pp. 61-69.
- [12] J. Oberhammer, M. Tang, A.-Q. Liu, G. Stemme, Mechanically tri-stable, true single-pole-double-throw (SPDT) switches, Journal of Micromechanics and Microengineering 16 (2006) 2251-2258.
- [13] T. Pendleton, B. Jensen, Development of a tristable compliant mechanism, Proceedings of 12th IFToMM World Congress, June 18-21, 2007, Paper no. 835.
- [14] G. Chen, D.L. Wilcox, L.L. Howell, Fully compliant double tensural tristable micromechanisms (DTTM), Journal of Micromechanics and Microengineering 19 (2009) 025011.
- [15] G. Chen, Q.T. Aten, S. Zirbel, B.D. Jensen, L.L. Howell, A tristable mechanism configuration employing orthogonal compliant mechanisms, Journal of Mechanisms and Robotics 2 (2010) 014501.
- [16] J.S. Han, C. Müller, U. Wallrabe, J.G. Korvink, Design, simulation, and fabrication of a quadstable monolithic mechanism with x- and y-directional bistable curved beams, Journal of Mechanical Design 129 (2007) 1198-1203.

- [17] Y.S. Oh, S. Kota, Synthesis of multistable equilibrium compliant mechanism using combinations of bistable mechanisms, Journal of Mechanical Design 131 (2009) 021002.
- [18] M. Hafez, M.D. Lichter, S. Dubowsky, Optimized binary modular reconfigurable robotic devices, IEEE/ASME Transactions on Mechatronics 8 (2003) 18-25.
- [19] L.L. Howell, Compliant mechanisms, John Wiley & Sons Inc. New York, USA, 2002.
- [20] M. Vangbo, An analytical analysis of a compressed bistable buckled beam, Sensors and Actuators A 69 (1998) 212-216.
- [21] S. Park, D. Hah, Pre-shaped buckled-beam actuators: Theory and experiments, Sensors and Actuators A 148 (2008) 186-192.
- [22] J. Qiu, J.H. Lang, A.H. Slocum, A curved-beam bistable mechanism, Journal of Microelectromechanical Systems 13 (2004) 137-146.
- [23] P. Cazottes, A. Fernandes, J. Pouget, M.Hafez, Bistable buckled beam: Modeling of actuating force and experimental validations, Journal of Mechanical Design 131 (2009) 101001.
- [24] J.M. Gere, S.P. Timoshenko, Mechanics of Materisls, Second edition, Wadsworth Inc., Belmont, California, USA, 1984.
- [25] K. Deb, A. Pratap, S. Agarwal, T. Meyarivan, A fast and elitist multiobjective genetic algorithm: NSGA-II, IEEE Transactions on Evolutionary Computation 6 (2002) 182-197.
- [26] H.D. Hibbitt, B.I. Karlsson., E.P. Sorensen, *ABAQUS user manual*, Version 6-2, HKS Inc. Providence, RI, USA, 2001.

- [27] K.P. Larsen, A.A. Rasmussen, J.T. Ravnkilde, M. Ginnerup, O. Hansen, MEMS device for bending test: measurements of fatigue and creep of electroplated nickel, Sensors and Actuators A 103 (2003) 156-164.
- [28] F. Arai, On-chip robotics: Technical issues and future direction, 2010 IEEE International Conference on Robotics and Automation Workshop on Micro-Nano Robotics and New Evolution, Anchorage, Alaska, May 3-8, 2010.
- [29] O.C. Jeong, S. Konishi, Pneumatic gas regulator with cascaded PDMS seal valves, Sensors and Actuators A 143 (2008) 84-89.

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Fig. 14 f- δ curve and maximum stress versus displacement curve of the microscale version of the QM for (a) forward motion; (b) backward motion.

	<u> </u>	
Variables	Lower bound (mm)	Upper bound (mm)
L_1	20	60
h_1	4	10
<i>w</i> ₁	0.83	1.1
L_2	20	60
h_2	4	10
<i>w</i> ₂	0.83	1.1

Table 1. Lower and upper bounds on the design variables.

Table 2. The values of the design variables of the optimum design.

Variable	Values (mm)
L_1	47.71
h_1	5.48
w_1	0.85
L_2	33.28
h_2	4.79
<i>w</i> ₂	0.96

Table 3. The values of the design variables of the optimum design of the microscale QM.

	-
Variable	Values (µm)
L ₁	770.60
h_1	44.10
<i>w</i> ₁	5.60
L_2	864.10
h_2	53.80
	4.00



Fig. 1 A 'ball-on-the-hill' analogy for a QM, similar to a figure presented by Chen et al. [14].



Fig. 2 Operational principle of the QM.



Displacement, **b**

Fig. 3 A typical force versus displacement curve of the QM and the corresponding configurations at displacement a, displacement b, displacement c, and displacement d, shown in the inlets.



(b)

Fig. 4 (a) A schematic of a quarter model. (b) Dimensions of the guide beam and the shuttle mass.



Fig. 5 Flowchart of the optimization procedure.



Fig. 6 A mesh for the finite element model.



Fig. 7 Distribution of the population of several generations in the optimization process.



Fig. 8 (a) f- δ curve and maximum stress versus displacement curve for forward motion; (b) strain energy curve for forward motion; (c) f- δ curve and maximum stress versus displacement curve for backward motion; (d) strain energy curve for backward motion.



Fig. 9 Photos of a fabricated QM.



Fig. 10 A photo of the experimental setup.



Fig. 11 Snapshots for forward motion (a-c) and backward motion (d-f).



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